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DESY

Particles, Strings,
and the Early Universe
Collaborative Research Center SFB 676

$\mathcal{N} = \frac{1}{2}$ domain walls & Yang-Mills instantons in heterotic supergravity

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CY 4-folds:	arXiv:1303.1832, arXiv:1405.2073 with James Gray and Andre Lukas
Main part:	arXiv:1202.5046, arXiv:1409.0548 with Karl-Philip Gemmer, Olaf Lechtenfeld, Edvard Musaev, Christoph Nölle, Alexander D. Popov

Plan

1. Brief advert for recent classification result for certain family of Calabi-Yau 4-folds
2. Main part
   - $\mathcal{N} = \frac{1}{2}$ domain wall solutions of heterotic supergravity
   - Lift to $\mathcal{O}(\alpha')$ and appearance of higher-dim. YM instantons
3. Conclusions

Calabi-Yau 4-folds

Motivation

- CY 4-folds are **important objects in string theory**. E.g.
  - to build $\mathcal{N} = 1$, $d = 4$ string vacua based on **F-theory**
  - **dualities** (M $\Longleftrightarrow$ F, het. ST $\Longleftrightarrow$ F)
  - **mirror symmetry**
- Our goal: systematically explore & map out **F-CY$_4$ "landscape"** (as opposed to "geometric engineering")
- Start: find description of all CY 4-folds which are **complete intersections in products of projective spaces (CICYs)**
  - Arguably "simplest" explicit CY constructions $\Longrightarrow$ many properties relatively straightforward to compute
  - Analog of very useful list of 7890 CICY 3-folds [Hübsch (1987); Green et. al. (1987); Candelas et. al. (1988)]
  - Generating equivalent 4-fold list requires qualitative and quantitative modifications ...

Example

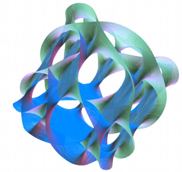
- An **example** of a CICY 4-fold configuration matrix (id 244)

$$\left[\begin{array}{c|cc} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 0 & 4 \end{array}\right]$$

  - Represents a family of CY 4-folds defined by the solutions to the polynomials

$$p_1 = \sum_{i,a} c_{ia} x^i y^a \ , \qquad p_2 = \sum_{i,\ldots,\delta} d_{iab\alpha\beta\gamma\delta} x^i y^a y^b z^\alpha z^\beta z^\gamma z^\delta$$

  - in the ambient space $\mathbb{P}^1 \times \mathbb{P}^2 \times \mathbb{P}^3$
- Simplest case: **“sextic”** [5|6] (4d analog of famous “quintic”)



Generalities

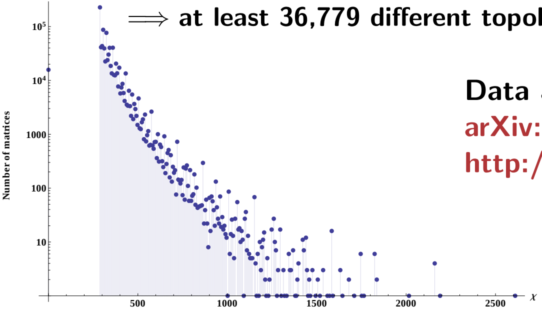
- (Family of) CICYs described by **configuration matrix**:

$$[\mathbf{n}|\mathbf{q}] \equiv \left[\begin{array}{c|ccc} n_1 & q_1^1 & \dots & q_K^1 \\ \vdots & \vdots & \ddots & \vdots \\ n_m & q_1^m & \dots & q_K^m \end{array}\right]$$

  - Ambient space: $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$
  - CICY: common zero locus of homogeneous polynomials $\{p_\alpha\}_{\alpha=1\ldots K}$
- Many **properties** of manifold **encoded** just in $[\mathbf{n}|\mathbf{q}]$, e.g.
  - Dimension of the complete intersection: $\sum_r n_r - K \overset{!}{=} 4$
  - Configuration is Calabi-Yau ($c_1 = 0$) if: $\sum_{\alpha=1}^{K} q_\alpha^r = n_r + 1$

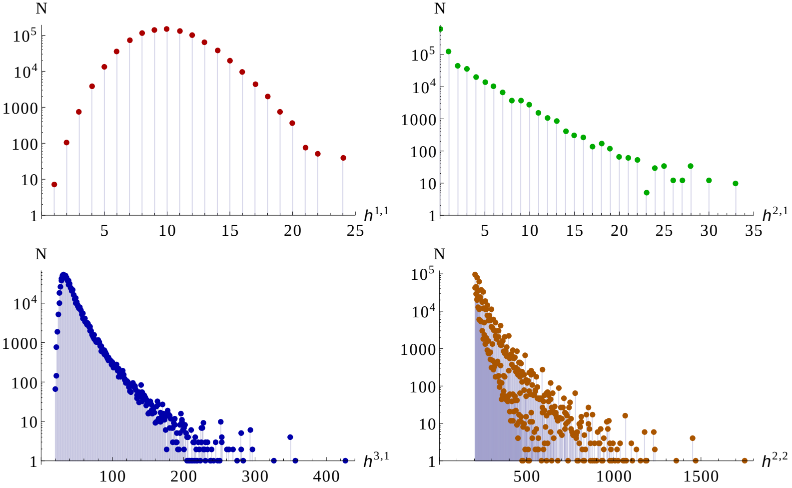
Results

- **Complete classification** ($\sim$ 1 yr. on large computer cluster)
  - Result: list of **921,497** configuration matrices
  - Same code correctly reproduces old CICY 3-fold list
  - Counting of product manifolds ($T^2 \times CY_3$, $T^4 \times K3$, T^8, $K3 \times K3$) comes out correctly
- Some redundancies are still present
  - distinguish via topological invariants
    $\Longrightarrow$ **at least 36,779 different topologies**



Data and code in arXiv:1303.1832 or at http://cicy4folds.haupt1.de

Hodge data


N
10^5
10^4
1000
100
10
1
5
10
15
20
25
$h^{1,1}$
N
10^5
10^4
1000
100
10
1
5
10
15
20
25
30
35
$h^{2,1}$
N
10^4
1000
100
10
1
100
200
300
400
$h^{3,1}$
N
10^5
10^4
1000
100
10
1
500
1000
1500
$h^{2,2}$

Elliptic Fibrations

- F-theory usually defined on **elliptically fibered** CY 4-fold
- No known general (& for our purpose practical) criterion for when 4-fold is elliptically fibered, but …
- for CICYs, $\exists$ **"obvious elliptic fibration" (=OEF)** struct.:

T^2

$$\left[\begin{array}{c|cc} 2 & 0 & 3 \\ 4 & 2 & 3 \end{array}\right]$$

base $[4|2]$

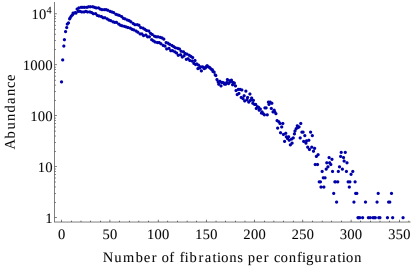
- More generally:

$$\left[\begin{array}{c|c} \mathcal{A}_1 & \mathcal{F} \end{array}\right] = T^2$$

Base: $\left[\begin{array}{c|c} \mathcal{A}_2 & \mathcal{B} \end{array}\right]$ $\quad \left[\begin{array}{c|cc} \mathcal{A}_1 & 0 & \mathcal{F} \\ \mathcal{A}_2 & \mathcal{B} & \mathcal{T} \end{array}\right]$

- Scan of all CICY 4-folds: **99.95% are OEF** ($\sim 1/2$ yr. on computer cluster)

Elliptic Fibrations — continued



- In total: $\sim \mathbf{5 \times 10^7}$ **fibrations**
- All manifolds with $h^{1,1} > 12$ have at least one such fibration (cf. similar observation for ell. fib. CY 3-folds [Taylor (1205.0952); Johnson, Taylor (1406.0514)])
- Can test some necc. conditions for **section** to exist (existence as a generic element of "favourable" divisor i.e. descending from hyperplanes in the ambient space)
  Computer scan: $\sim \mathbf{2.6 \times 10^7}$ **($\sim$ 52%) pass test**

$\mathcal{N} = \frac{1}{2}$ domain wall solutions of heterotic supergravity

Motivation

- One classic route to $\mathcal{N} = 1$, $d = 4$ physics from string theory is $\mathcal{M}^{10} = \mathbb{R}^{1,3} \times \mathrm{CY}_3$, but **difficult to stabilize all moduli**
- Better: $\mathrm{CY}_3 \rightarrow$ more general **SU(3) structure mfld** (+ flux)
- But then: in general, no perturbative $\mathcal{N} = 1$ Minkowski vacua exist (except "Strominger system")
- Phenomenologically: want vacuum (close to) Minkowski **after** taking into account perturbative and non-perturbative effects
- **Non-perturbative corrections** to superpotential required in many moduli stabilization scenarios $\rightarrow$ too restrictive to demand existence of Minkowski vacuum in their absence
- E.g.

$$\boxed{\text{Pert. } \mathcal{N} = 1/2 \text{ DW}} + \boxed{\text{non-pert. effects}} \rightarrow \boxed{\mathcal{N} = 1 \text{ Mink.}}$$

such as gaugino condensation, membrane instantons

(or close to)

- Balancing – if possible – requires **fine tuning** similar to KKLT (PE anomalously small) or LVS (NPE anomalously large)

Heterotic supergravity

- **Ingredients** on $d=10$ manifold $\mathcal{M}$:
  - Lorentzian metric $\hat{g}$
  - Neveu-Schwarz 3-form $\hat{H} \in \Omega^3(\mathcal{M})$
  - dilaton $\hat{\phi} : \mathcal{M} \to \mathbb{R}$
  - gauge connection ${}^A\hat{\nabla}$ with gauge group $SO(32)$ or $E_8 \times E_8$
- Anomaly cancellation condition leads to **Bianchi identity**:

$$\hat{\mathrm{d}}\hat{H} = \frac{\alpha'}{4}\mathrm{Tr}(\hat{F}\wedge\hat{F} - \tilde{R}\wedge\tilde{R})$$

at $\mathcal{O}(\alpha')$ (here: $\tilde{R}\cdot\epsilon = 0$). At lowest order: $\hat{\mathrm{d}}\hat{H} = 0$.

- **BPS equations** (SUSY background):
  - gravitino: ${}^-\hat{\nabla}\epsilon = 0$
  - dilatino[1]: $\left(\hat{\mathrm{d}}\hat{\phi} - \frac{1}{2}\hat{H}\right)\cdot\epsilon = 0$

  $\Big\} \Longrightarrow$ G structure
  - gaugino[1]: $\hat{F}\cdot\epsilon = 0$ $\Longrightarrow$ (later: YM instantons)

[1] Def$^{\text{n}}$: $\omega\cdot\epsilon = \frac{1}{p!}\omega_{i_1\dots i_p}\gamma^{i_1}\cdots\gamma^{i_p}\epsilon$, where γ^i are Clifford matrices ($\{\gamma^i,\gamma^j\} = 2g^{ij}$)

Ansatz

- Let's consider $\mathcal{M}=\mathbb{R}^{1,2}\times\mathbb{R}\times X_6$, with X_6 compact
- **Metric**
  $\hat{g}=\mathrm{e}^{2A(x^m)}\left(\eta_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta+\mathrm{e}^{2\Delta(x^u)}\mathrm{d}x^3\mathrm{d}x^3+g_{uv}(x^m)\mathrm{d}x^u\mathrm{d}x^v\right)$
- $d=1+2$ **domain wall**. WV: $\{x^\alpha\}$, trans.: $\{x^m\}=\{x^3,x^u\}$
- Killing spinor: $\epsilon(x^\alpha,x^m)=\rho(x^\alpha)\otimes\eta(x^m)\otimes\theta$
- ρ has two real components $\Longrightarrow$ our background preserves **two real supercharges** ($\mathcal{N}=\frac{1}{2}$ SUSY in $d=4$ terminology)
- Lorentz invariance on $d=1+2$ domain wall world-volume $\Longrightarrow\ \partial_\alpha\hat{\phi}=0,\ \hat{H}_{\alpha mn}=0,\ \hat{H}_{\alpha\beta n}=0$
- Simplification in this talk: $A=0,\ \Delta=0,\ \hat{H}_{\alpha\beta\gamma}=0$

[Lukas, Matti (1005.5302); Gray, Larfors, Lüst (1205.6208);
AH, Lechtenfeld, Musaev (1409.0548)]

G_2 structure on $X_7 = \mathbb{R} \times X_6$

- Consider $d = 7$ part of previous ansatz:
  $X_7 = \mathbb{R} \times X_6, \quad g_7 = \mathrm{d}x^3\mathrm{d}x^3 + g_{uv}(x^m)\mathrm{d}x^u\mathrm{d}x^v$
- On X_7 we can construct a **globally well-defined 3-form** $\varphi \in \Omega^3(X_7)$ and its $d = 7$ Hodge dual $\Phi := *_7\varphi \in \Omega^4(X_7)$

$$\varphi_{mnp} = -\mathrm{i}\eta^\dagger\Gamma_{mnp}\eta \ , \qquad \Phi_{mnpq} = \eta^\dagger\Gamma_{mnpq}\eta \ .$$

  with $\{\Gamma_m, \Gamma_n\} = 2(g_7)_{mn}$ and $\Gamma_{m_1\ldots m_p} := \Gamma_{[m_1}\cdots\Gamma_{m_p]}$.
- First two BPS equations then imply

$$\mathrm{d}_7\varphi = 2\mathrm{d}_7\hat{\phi}\wedge\varphi - *_7\hat{H} \ , \qquad *_7\mathrm{d}_7\hat{\phi} = -\tfrac{1}{2}\hat{H}\wedge\varphi \ ,$$
$$\mathrm{d}_7\Phi = 2\mathrm{d}_7\hat{\phi}\wedge\Phi \ , \qquad 0 = \hat{H}\wedge\Phi \ .$$

- **First two BPS equations** $\Longrightarrow$ G_2 **structure on** X_7

$SU(3)$ structure on X_6

- **Decompose** η into two $d = 6$ spinors of definite chirality:

$$\eta = \frac{1}{\sqrt{2}}(\eta_+ + \eta_-)$$

- They still depend on non-compact transverse direction $y := x^3$
- **For every fixed value of** y, have globally well-defined real 2-form J and complex 3-form $\Omega = \Omega_+ + \mathrm{i}\Omega_-$

$$\Omega_{uvw} = \eta_+^\dagger \gamma_{uvw} \eta_- \ , \qquad J_{uv} = \mp \eta_\pm^\dagger \gamma_{uv} \eta_\pm \ .$$

- Specifies a **static SU(3) structure** on X_6
- G_2 structure on $X_7 \leftrightarrow SU(3)$ structure on X_6

$$\varphi = \mathrm{d}y \wedge J + \Omega_- \ , \qquad \Phi = \mathrm{d}y \wedge \Omega_+ + \tfrac{1}{2} J \wedge J \ .$$

Zeroth order nearly Kähler solutions

- BI @ $\mathcal{O}((\alpha')^0)$ is simply $\hat{\mathrm{d}}\hat{H} = 0$
- Gauge sector trivial (only condition $\hat{F} \cdot \epsilon = 0$, solved by $\hat{F} = 0$)
- Restrict $SU(3)$ structure to **nearly Kähler** (torsion class W_1)

$$\mathrm{d}J = -\tfrac{3}{2} W_1^- \Omega_+ + \tfrac{3}{2} W_1^+ \Omega_- \ , \quad \mathrm{d}\Omega = W_1 J \wedge J \ .$$

- **BPS** + **BI** $\implies$ $\hat{H} = -\frac{1}{2}\phi' \Omega_+ + \frac{3}{2} W_1^- \Omega_- - 2 W_1^- J \wedge \mathrm{d}y$ and

$$\begin{aligned} 0 &= \phi' W_1^+ - 3(W_1^-)^2 \ , \\ 0 &= \phi'' + \tfrac{3}{2}(\phi')^2 + \tfrac{13}{2}\phi' W_1^+ \ , \\ 0 &= \phi'\alpha - 3(W_1^-)' - \tfrac{21}{2} W_1^+ W_1^- - \tfrac{9}{2}\phi' W_1^- \ . \end{aligned}$$

- **2 special solutions:** [Lukas et.al. (1005.5302); Lüst et.al. (1205.6208)]
  - NK with $\phi = \text{const.}$, $\hat{H} = 0$ ($W_1^+ = \text{any}$, $W_1^- = 0$, $\alpha = \text{any}$)
  - CY with flux ($\phi = \frac{2}{3}\log(ay+b)$, $W_1 = 0$, $\alpha = 0$)

Lift to $\mathcal{O}(\alpha')$ and appearance of higher-dim. YM instantons

Appearance of higher-dim. YM instantons

- Need to deal with **non-trivial BI**:

$$\hat{\mathrm{d}}\hat{H} = \frac{\alpha'}{4}\mathrm{Tr}(\hat{F} \wedge \hat{F} - \tilde{R} \wedge \tilde{R})$$

- $\overset{\text{in general}}{\Longrightarrow}$ $\hat{F} \neq 0$ and thus need to solve

$$\hat{F} \cdot \epsilon = 0$$

- Solutions to $\hat{F} \cdot \epsilon = 0$ are called **YM instantons**
- Originally YM instantons first appeared in a different context ...

Yang-Mills instantons in $d = 4$

Definition

A (classical) Yang-Mills instanton is a gauge connection on Euclidean $\mathcal{M}^4$, whose curvature F is **self-dual**, i.e. $*F = F$.

Properties

- **Solutions of YM-eq.** $(0 \overset{\text{BI}}{=} DF = D * F \implies D * F = 0)$
- **Absolute minima** of YM-action $S_{\text{YM}} = \frac{1}{2e} \int_{\mathcal{M}^4} \text{Tr}(F \wedge *F)$ within their topological type
- 1st order eq. **easier to solve** than 2nd order YM-eq.
- **Localized in space & time** (when Wick rotated $\mathcal{M}^4 \to \mathcal{M}^{1,3}$), hence called "instanton" or "pseudoparticle"

[Belavin, Polyakov, Schwarz, Tyupkin (1975); Atiyah, Hitchin, Singer (1977); Atiyah, Drinfeld, Hitchin, Manin (1977), ...]

Applications & Example

- Maths: **classification of 4-manifolds** (e.g. Donaldson invariants [Donaldson (1983)])
- Physics: ['t Hooft (1976); Jackiw, Rebbi (1976); Callan et.al. (1978), ...]
  - Instantons are critical points of YM-action
  - Appear in PI as leading quantum corrections to class. behavior
  - $\Longrightarrow$ learn about **structure of YM-vacuum**
- 1st example: **BPST inst.** [Belavin, Polyakov, Schwarz, Tyupkin (1975)]

$\mathcal{M}^4=\mathbb{R}^4$, $G=SU(2)$, $x^\mu=(\{x^i\}_{i=1,2,3},x^4)$

$$A_\mu=\frac{r^2}{r^2+\lambda^2}g^{-1}(x)\partial_\mu g(x)\,,$$

with $r^2:=\sum_\mu(x^\mu)^2$, $g(x)=\frac{x^4+\mathrm{i}\sum_i x^i\sigma^i}{r}$, $\sigma^i=$ Pauli matrices.

Note: $S_{\mathrm{YM}}=\frac{8\pi^2}{e^2}$. PI $\mathrm{e}^{-S_{\mathrm{YM}}}=\mathrm{e}^{-(8\pi^2)/e^2}$
(no pert. exp. in e $\Longrightarrow$ non-pert.)

Yang-Mills instantons in $d > 4$

Definition

In higher dimensions, the instanton equation is generalized to

$$*F = -(*Q) \wedge F \, ,$$

with some globally well-defined 4-form Q.

[Corrigan, Devchand, Fairlie, Nuyts (1983)]

Properties

- Need additional structure on $\mathcal{M}$ to have $Q \leftrightarrow G$ **structure**
- Instanton eq. $\implies$ **YM with torsion** $D * F + F \wedge *H = 0$. Torsion 3-form $*H := d * Q$ (ordinary YM if Q co-closed).
- Action for YM with torsion is **finite** for instantons $S_{\text{YM}} + S_{\text{CS}} = \int_{\mathcal{M}} \text{Tr}\left\{F \wedge *F + (-1)^{d-3} F \wedge F \wedge *Q\right\} < \infty$
- Torsion appears naturally in string theory ($H \leftrightarrow$ NS flux)

Yang-Mills instantons in $d > 4$ — continued

Alternative definitions

- $F \cdot \epsilon = 0$ (the defn I gave earlier!)
- $F \in \mathfrak{g}$ (often in math. lit., $\mathfrak{g} =$ Lie alg. of structure group G)
- In general: $F \cdot \epsilon = 0 \implies F \in \mathfrak{g} \implies *F = -(*Q) \wedge F$

- Which manifolds admit a globally well-defined 4-form Q?
  $\rightarrow$ G **structure manifolds** (i.e. manifolds with weak special holonomy), e.g. [Berger (1955); Bär (1993)]
  - $SU(3)$ structure in $d = 6$
  - G_2 structure in $d = 7$
  - Spin(7) structure in $d = 8$

Example

Instanton solution on (cylinder over) compact nearly Kähler X_6

- $SU(3)$ structure on X_6 characterized by globally well-defined **real 2-form** J and **complex 3-form** $\Omega = \Omega_+ + \mathrm{i}\Omega_-$

$$\mathrm{d}J = -\tfrac{3}{2}\,\mathrm{Im}(W_1\bar{\Omega}) + W_4 \wedge J + W_3 \,,$$
$$\mathrm{d}\Omega = W_1 J \wedge J + W_2 \wedge J + \bar{W}_5 \wedge \Omega \,.$$

- **5 intrinsic torsion classes**, e.g.
  - $W_1 = \ldots = W_5 = 0 \Leftrightarrow \mathrm{CY}_3$ ($\mathrm{d}J = 0$ & $\mathrm{d}\Omega = 0$)
  - $W_2 = \ldots = W_5 = 0 \Leftrightarrow$ nearly Kähler
- Now, specialize to **NK** X_6 with $W_1^+ = 2$, $W_1^- = 0$, i.e.

$$\mathrm{d}J = 3\Omega_- \,, \quad \mathrm{d}\Omega_+ = 2J \wedge J \,.$$

- Examples: **4 homogeneous spaces** [Butruille (math/0612655)]
$S^6 = G_2/SU(3)$, $\mathbb{F}^3 = SU(3)/U(1)^2$,
$\mathbb{CP}^3 = Sp(2)/Sp(1) \times U(1)$, $S^3 \times S^3 = SU(2)^3/SU(2)_{\mathrm{diag}}$

Example — continued

- On X_6 always **2 real Killing spinors** $\eta_\pm$ of opposite chirality

$$ {}^{\mathrm{LC}}\nabla_u \eta_\pm = \mp \tfrac{\mathrm{i}}{2} \gamma_u \eta_\pm \ . $$

- **Important observation:** $P_{uvw} \gamma^{vw} \eta_\pm = 4\mathrm{i} \gamma_u \eta_\pm$ (with $P := \Omega_+$)

Definition

Connection ${}^{\mathrm{can}}\nabla$ on G structure manifold is **canonical** if it has holonomy G and torsion totally anti-symmetric w.r.t. some G-compatible metric.

- Here: ${}^{\mathrm{can}}\Gamma^w_{uv} = {}^{\mathrm{LC}}\Gamma^w_{uv} + \frac{1}{2} P_{uvw}$ $({}^{\mathrm{can}}\nabla_u v^v = \partial_u v^v + {}^{\mathrm{can}}\Gamma^v_{uw} v^w)$
- $\begin{cases} {}^{\mathrm{can}}\nabla \eta_\pm = 0 & (\mathrm{Hol}({}^{\mathrm{can}}\nabla) = SU(3); \text{note: } {}^{\mathrm{LC}}\nabla \eta_\pm \neq 0) \\ T^u = \frac{1}{2} P^u{}_{vw} e^v \wedge e^w & \end{cases}$

Example — continued

Back to instanton equation:

- What is Q? $\rightarrow$ $Q = *J = \frac{1}{2} J \wedge J$
- $*F = -J \wedge F \Leftrightarrow (F \in \Omega^{1,1}$ and $J \lrcorner F = 0)$ **(DUY; herm. YM)**
- **Important:** ${}^{\mathrm{can}}\nabla$ is an instanton on X_6 [Harland, Nölle (1109.3552)]
- Consider **cylinder over NK coset**, $Z(K/H) = \mathbb{R} \times (K/H)$, with metric $g_7 = \mathrm{d}\tau \otimes \mathrm{d}\tau + g_6$
- $SU(3)$ structure on K/H lifts to G_2 structure on $Z(K/H)$

$$\varphi = \mathrm{d}\tau \wedge J + \Omega_- \ , \quad \Phi := *_7 \varphi = \mathrm{d}\tau \wedge \Omega_+ + \tfrac{1}{2} J \wedge J \ .$$

- In general, G_2 structure equations:

$$\begin{aligned} \mathrm{d}_7 \varphi &= \tau_0 \Phi + 3\tau_1 \wedge \varphi + *_7 \tau_3 \ , \\ \mathrm{d}_7 \Phi &= 4\tau_1 \wedge \Phi + \tau_2 \wedge \varphi \ . \end{aligned}$$

- Here: **(loc.) conformally parallel G_2 structure** $(\tau_1 = -\mathrm{d}\tau)$

Example — continued

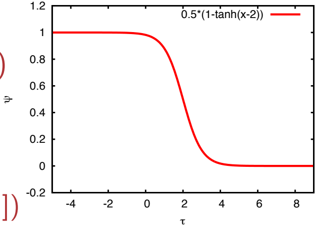
[Harland, Nölle (1109.3552); AH, Lechtenfeld, Musaev (1409.0548)]

- Recall: **coset** $X_6 = K/H$
- Lie algebra decomposes: $\mathfrak{k} = \mathfrak{h} \oplus \mathfrak{m}$
- Lie algebra generators of $\mathfrak{k}$ split: $\{I_A\} = \{I_i\} \cup \{I_u\}$
- **Ansatz** for instanton connection on $Z(K/H)$

$$^{A}\nabla = {}^{\text{can}}\nabla + \psi(\tau) e^u I_u \ ,$$

parametrized by **one function** $\psi(\tau)$.

- Instanton equation $\implies$ $\dot{\psi} = 2\psi(\psi - 1)$
- **Solutions:**
  - $\psi = 0$ $\quad [{}^{A}\nabla = {}^{\text{can}}\nabla]$
  - $\psi = 1$ $\quad [{}^{A}\nabla = \text{LC-conn. of cone}]$
  - **Tanh-kink**: $\psi(\tau) = \frac{1}{2}\left(1 - \tanh[\tau - \tau_0]\right)$



NK DW solutions @ $\mathcal{O}(\alpha')$

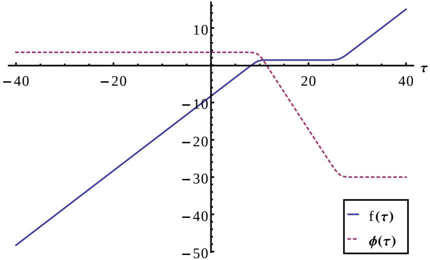
[Harland, Nölle (1109.3552); Lukas et.al. (1210.5933); AH et.al. (1202.5046, 1409.0548)]

- **Ansatz:** $\mathcal{M} = \mathbb{R}^{1,2} \times \mathbb{R} \times X_6$

$$\hat{g} = \eta_{\alpha\beta} \mathrm{d}x^\alpha \mathrm{d}x^\beta + \mathrm{e}^{2f(\tau)} \left(\mathrm{d}\tau^2 + g_{uv}(x^w) \mathrm{d}x^u \mathrm{d}x^v\right) \ ,$$

$$^{-}\hat{\nabla} = {}^{\mathrm{can}}\nabla + \psi_1(\tau) e^u I_u \ ,$$

$$^{A}\hat{\nabla} = {}^{\mathrm{can}}\nabla + \psi_2(\tau) e^u I_u \ .$$



- **Solution:**

$$\hat{H} = -\frac{\alpha'}{4} \left(\psi_1^2(2\psi_1 - 3) - \psi_2^2(2\psi_2 - 3)\right) \Omega_+ \ ,$$

$$\phi = \phi_0 + 2(f - \tau) \ ,$$

$$\mathrm{e}^{2f} = \mathrm{e}^{2(\tau - \tau_0)} + \frac{\alpha'}{4} \left(\psi_1^2 - \psi_2^2\right) \text{ with } \psi_{1,2} \in \{0, 1, \text{tanh-kink}\}.$$

- **8 distinct cases**, e.g. $\psi_1 = 1$, $\psi_2 =$ tanh-kink

Conclusions

Summary

- $\boxed{\text{Pert. } \mathcal{N} = 1/2 \text{ DW}} + \boxed{\text{non-pert. effects}} \to \boxed{\mathcal{N} = 1 \text{ Mink.}}$
- First two BPS eqs. $\implies$ $SU(3)$ str. on X_6, G_2 str. on $\mathbb{R} \times X_6$
- @ $\mathcal{O}((\alpha')^0)$: $\hat{F} = 0$ and e.g. (NK with $\phi = \text{const.}$, $\hat{H} = 0$) or (CY w/ flux)
- @ $\mathcal{O}(\alpha')$: $\hat{F} \cdot \epsilon = 0 \implies$ **higher-dim. YM instantons**
- Find explicit instantons on $\mathbb{R}\times$NK which have **tanh-kink shape** and are useful for constructing full $\mathcal{O}(\alpha')$ **heterotic domain wall solutions**

Outlook

- Recently obtained new instantons (and non-instanton YM configs) on $\mathbb{R} \times X_7$, where X_7 has G_2 str. or even $SU(3)$ str. (to appear soon)
- Qu.: embed into het. SUGRA as new (non-)SUSY solns.?

Thank you for your attention.

Backup slides

More about sections

Dynamic $SU(3)$ structure on X_6

- From first two BPS eqs. (restore y-dependence of J, Ω):

$$\mathrm{d}J = \Omega'_- - 2\phi'\Omega_- - *H\,, \qquad 0 = *\phi' + \tfrac{1}{2}H\wedge\Omega_-\,,$$
$$J\wedge \mathrm{d}J = 0\,, \qquad 0 = \tfrac{1}{2}H_y\wedge\Omega_- - \tfrac{1}{2}H\wedge J\,,$$
$$\mathrm{d}\Omega_+ = J\wedge J' - \phi' J\wedge J\,, \qquad 0 = \Omega_+\wedge H + \tfrac{1}{2}H_y\wedge J\wedge J\,,$$
$$\mathrm{d}\Omega_- = -*H_y\,,$$

with $\mathrm{d}_7\omega = \mathrm{d}\omega + \mathrm{d}y\wedge\omega'$, $\hat\phi = \phi(y)$ and $\hat H = H + \mathrm{d}y\wedge H_y$.

- **Generalization of Hitchin flow equations for dynamic $SU(3)$ structure** — common in $d = 4$ BPS DW solutions of $d = 10$ SUGRA theories [Mayer, Mohaupt (hep-th/0407198); Louis, Vaulà (hep-th/0605063); Smyth, Vaulà (0905.1334)]
- For $\hat H = 0$ and $\phi = 0$, restore original Hitchin flow equations:

$$J\wedge\mathrm{d}J = 0\,, \qquad \mathrm{d}J = \Omega'_-\,,$$
$$\mathrm{d}\Omega_- = 0\,, \qquad \mathrm{d}\Omega_+ = J\wedge J'\,.$$